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Non-radiative surface plasmon-polariton modes of polar semiconductor cylinders

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Abstract : The dispersion relation for surface plasmon-polariton modes of polar semiconductor cylinders has been derived using Bloch hydrodynamic method and the radius dependence of the non-radiative modes has been studied. For small values of cylinder a minimum is observed in the curve The inclusion of spatial dispersion results in finite life time of the surface polariton modes even in collisionless plasma system.

Keywords : Plasmon-polariton interaction, polar semiconductor cylinders, dispersion relation

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1. Introduction

The study of the properties of surface polaritons has recently received considerable attention [1]. Surface polaritons are the coupled photon and elementary excitation modes localized at the surface or bound to the interface of dielectric or magnetic media [2], and are extremely sensitive probes for the surface properties as they have the maximal electromagnetic (EM) field intensity just on the surface. These studies are of wide applications in the field of integrated optics, microelectronics *etc.* [3]. Physical properties of the media near the surface differ from bulk properties and the information on these properties can be obtained by studying the dispersion relation for surface polariton modes.

The properties of surface modes depend upon the nature and geometry of the guiding surface. In planar or spherical surfaces, transverse magnetic (TM) or transverse electric (TE) modes may exist independently [4], but in cylindrical surfaces, independent existence

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of TM or TE modes is not possible in general, hence mixed modes have to be used in order to satisfy the boundary conditions at the surface of cylinder [5]. The study of behaviour of surface modes in materials for cylindrical geometry is of great scientific and practical importance, especially in the field of fibre optics [6].

In the present paper, a study of surface plasmon-polariton modes of polar semiconductor cylinder has been taken up using the Bloch hydrodynamical method [7] to incorporate spatial dispersion effects. It provides a simple, yet effective way to introduce the effects of spatial dispersion in the study, without using the non-local dielectric function, which avoids complicated mathematical analysis [8,9]. This method has been successfully employed by several workers [9–11] for the study of surface plasmon-polariton modes in spatially dispersive metallic plasma systems. Srivastava *et al* [12] modified this method to study the coupling of surface plasmon (SP) and surface optical phonon (SOP) modes of polar semiconductors. This modified Bloch's hydrodynamic method has been utilized here to study the surface plasmon-polariton modes of a polar semi-conductor cylinder.

2. Dispersion relation

The Bloch's hydrodynamic equations for a collisionless polar semiconductor can be solved using the perturbation method. The first order perturbation terms, on coupling with Maxwell's equations, yield [13]

$$(\beta^2\nabla^2 + \omega^2 - \omega_p^2)\nabla.\bar{E} = 0 \tag{1}$$

and

$$\left[c^2\nabla^2 + \bar{\varepsilon}(\omega^2 - \omega_p^2)\right]\nabla \times \bar{E} = 0, \tag{2}$$

where $\bar{\varepsilon}$ is the frequency-independent approximation of lattice dielectric function, and

$$\varepsilon_L(\omega) = \frac{\varepsilon_\infty\omega^2 - \varepsilon_0\omega_t^2}{\omega^2 - \omega_t^2}, \tag{3}$$

where ε_∞ and ε_0 are the high and low frequency dielectric constants and ω_t is the transverse optical phonon frequency. $\bar{\varepsilon}$ is given by [12]

$$\bar{\varepsilon} = \frac{\varepsilon_0 + \varepsilon_\infty}{2}. \tag{4}$$

ω_p in eqs. (1) and (2) is the bulk plasma frequency and is given by

$$\omega_p = \left(\frac{4\pi n_{e0}e^2}{\bar{\varepsilon}m}\right)^{1/2}, \tag{5}$$

where n_{e0} is the equilibrium electron density. Spatial dispersion is included through the quantity β, which is the average speed of propagation of disturbance through the electron gas, where [14]

$$\beta = \sqrt{\tfrac{3}{5}}\, V_F, \tag{6}$$

V_F being the Fermi velocity. The time dependence of the EM fields is assumed to be $e^{-i\omega t}$.

Eqs. (1) and (2) may be written as :

$$[\nabla^2 - \gamma^2]\nabla\cdot\bar{E} = 0 \tag{7}$$

and
$$[\nabla^2 - \alpha^2]\nabla\times\bar{E} = 0, \tag{8}$$

where
$$\gamma^2 = \frac{\omega_p^2 - \omega^2}{\beta^2} \tag{9}$$

and
$$\alpha^2 = \frac{\bar{\varepsilon}(\omega_p^2 - \omega^2)}{c^2}. \tag{10}$$

Eqs. (7) and (8) yield an irrotational solution $\bar{E}^{(\gamma)}$ and divergence-free solutions $\bar{E}_1^{(\alpha)}$ and $\bar{E}_2^{(\alpha)}$. These solutions can be expressed in terms of scalar functions ϕ_γ and ϕ_α as [15]

$$\bar{E}^{(\gamma)} = \nabla\phi_\gamma, \tag{11}$$

$$\bar{E}_1^{(\alpha)} = \nabla\times(\hat{r}\,\bar{r}\,\phi_\alpha), \tag{12}$$

and
$$\bar{E}_2^{(\alpha)} = \frac{1}{\alpha}\left(\nabla\times\bar{E}_1^{(\alpha)}\right), \tag{13}$$

where $\hat{r}$ is a unit vector along $\bar{r}$.

The system under consideration consists of a homogeneous, isotropic, non-magnetic polar semiconductor circular cylinder of radius R, its axis coinciding with the z-axis and embedded in a non-dispersive isotropic dielectric medium. The polar semiconductor is characterized by frequency dependent dielectric function $\varepsilon(\omega)$ and the bounding medium by dielectric constant ε_B.

The scalar functions ϕ_γ and ϕ_α can be expanded in terms of cylindrical coordinates as

$$\phi_\gamma = \exp(ikz)\sum_n\left[\{\exp(in\theta)\}\,\phi_{\gamma n}(r)\right] \tag{14}$$

and
$$\phi_\alpha = \exp(ikz)\sum\left[\{\exp(in\theta)\}\,\phi_{\alpha n}(r)\right] \tag{15}$$

where $\bar{k}$ is the component of wave vector along z-axis, and n is an integer denoting the mode number. The azimuthal dependence of the field enters through $e^{in\theta}$, the variation of the fields along the axis of the cylinder through e^{ikz} and the radial dependence involves various Bessel functions.

The use of eqs. (19) and (15) in eqs. (7) and (8) leads to the differential equations

$$\gamma^2\phi''_{\gamma n} + \phi'_{\gamma n} - (n^2 + k^2r^2 + \gamma^2r^2)\phi_{\gamma n} = 0 \tag{16}$$

and
$$\gamma^2\phi''_{\alpha n} + \phi'_{\alpha n} - (n^2 + k^2r^2 + \alpha^2r^2)\phi_{\alpha n} = 0, \tag{17}$$

where primes denote differentiations with respect to r. The solutions of the above two equations involve standard modified Bessel functions, defined as $I_n\ (\theta_\alpha r)$, $K_n\ (\theta_\alpha r)$ and $K_n\ (\theta_\gamma r)$ [16,17], where θ_α and θ_γ are given by

$$\theta_\alpha^2 = k^2 + \alpha^2 \tag{18}$$

and

$$\theta_\gamma^2 = k^2 + \gamma^2. \tag{19}$$

As the geometry under consideration is cylindrical, the total field will be a linear combination of longitudinal, TM and TE components [9].

The field outside the cylinder is obtained from the scalar function θ_δ in a similar manner, where

$$\delta^2 = -\varepsilon_B \frac{\omega^2}{c^2} \tag{20}$$

and

$$\theta_\delta^2 = k^2 + \delta^2. \tag{21}$$

The application of boundary conditions [9] *viz.* the continuity of E_θ, E_z, B_θ, B_z and D_r at the polar semiconductor dielectric interface *i.e.* at $r = R$, yields the required dispersion relation for surface plasmon-polariton modes, including spatial dispersion effects as :

$$\begin{vmatrix} -I_1' & -\frac{nk}{\alpha R} I_1 & K_1' & \frac{nK}{\delta R} K_1 & \frac{in}{R} I_2 \\ 0 & -\frac{\theta_\alpha^2}{\alpha} I_1 & 0 & \frac{\theta_\delta^2}{\delta} K_1 & iKI_2 \\ \frac{inK}{\Omega R} I_1 & \frac{i\alpha}{\Omega} I_1' & -\frac{inK}{\Omega R} K_1 & -\frac{i\delta}{\Omega} K_1' & 0 \\ \frac{i\theta_\alpha^2}{\Omega} I_1 & 0 & -\frac{i\theta_\delta^2}{\Omega} K_1 & 0 & 0 \\ \frac{in\bar{\varepsilon}}{R} I_1 & \frac{iK\bar{\varepsilon}}{\alpha} I_1' & -\frac{in\varepsilon_B}{R} K_1 & -\frac{in\varepsilon_B}{\delta} K_1' & \bar{\varepsilon} I_2 \end{vmatrix} = 0, \tag{22}$$

where $I_1 = I_n(\theta_\alpha R)$, $I_2 = I_n(\theta_\gamma R)$, $K_1 = K_n(\theta_\delta R)$

and I_1', I_2' and K_1' are their primes respectively and n is an integer designating the mode number, $\Omega\ (= \omega\ /\omega_p)$ and $K (= ck / \omega_p)$ are the dimensionless reduced frequency and wave-vector respectively. R is measured in terms of c/ω_p. I_n and K_n are standard modified Bessel functions, I_n' and K_n' are derivatives with respect to their arguments.

The dispersion relation for surface plasmon-polariton modes for a polar semiconductor cylinder, neglecting spatial dispersion effects can be obtained from eq. (22) by taking limit $\gamma R \to \infty$ (*i.e.* $\beta \to 0$), as

$$R^2\left[\alpha^2\theta_\delta^4\left\{\frac{I_n'(\theta_\alpha R)}{I_n(\theta_\alpha R)}\right\}^2+\delta^2\theta_\alpha^4\left\{\frac{K_n'(\theta_\delta R)}{K_n(\theta_\delta R)}\right\}^2\right.$$

$$\left.-\left(\alpha^2+\delta^2\right)\theta_\alpha^2\theta_\delta^2\,\frac{I_n'(\theta_\alpha R)}{I_n(\theta_\alpha R)}\cdot\frac{K_n'(\theta_\delta R)}{K_n(\theta_\delta R)}\right]$$

$$-\;n^2K^2\left(\theta_\alpha^2-\theta_\delta^2\right)^2\;=\;0. \tag{23}$$

The dispersion curves for surface plasmon-polariton modes without spatial dispersion [eq. (23)] have been shown in Figure 1 for InAs [18] cylinder taking the bounding medium to be vacuum ($\varepsilon_B = 1$) for $n = 1$ and $R = 10$ ($\sim 6.29 \times 10^{-3}$ cm), $R = 1$ ($\sim 6.29 \times 10^{-4}$ cm), $R = 0.01$ ($\sim 6.29 \times 10^{-6}$ cm). The uncoupled surface plasmon modes and the light line ($\Omega = K$) have also been plotted in the same figure.

In the present work, attention has been confined only to the study of non-radiative surface modes which lie in the region $\Omega < K$. A non-radiative surface polariton mode propogates along an interface without decay, but decays exponentially in a non-oscillatory manner in directions perpendicular to the interface. This type of mode is associated with $\varepsilon(\omega) < 0$ [13].

From Figure 1, it is observed that non-radiative surface plasmon-polariton mode tends towards the light line for low values of wave vector and for high values of wave

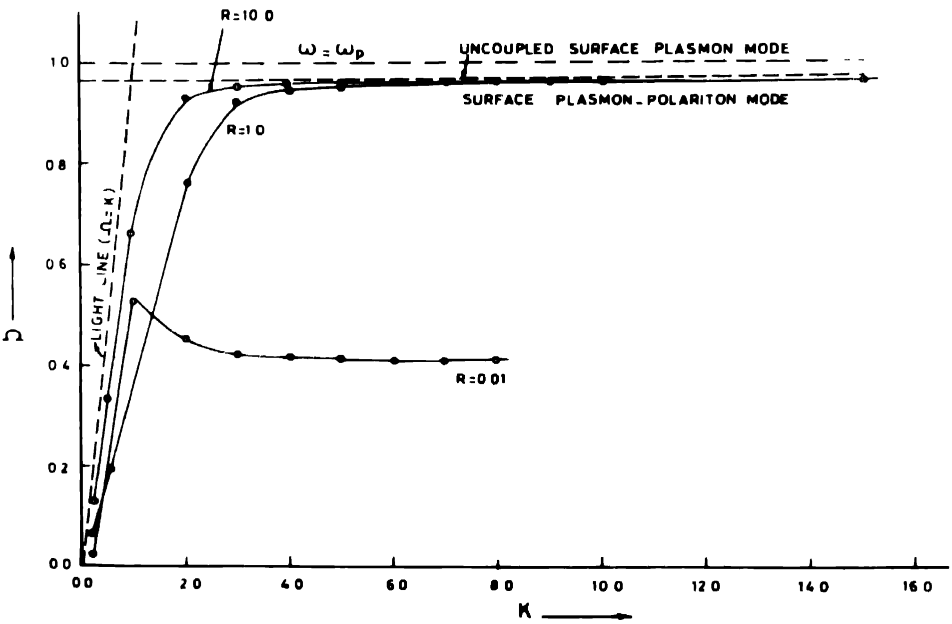

Figure 1. Dispersion curves for surface plasmon-polaritons without spatial dispersion for InAs cylinders of different radii ($n = 1$, $R = 10$ and 0.01) in vacuum

vector the coupled mode tends towards the uncoupled surface plasmon mode. Thus, it is photon-like for low wave vector values and surface plasmon-like for high wave vector values. For intermediate values of the wave vector, the coupled mode exhibits a mixed surface plasmon-photon character which is most prominent in the region where the uncoupled surface plasmon mode and the light line intersect each other. A minimum in the dispersion curve is observed for R = 0.01 (thin cylinder). The minimum is found to disappear with an increase in cylinder radius, and thus is a characteristic of thin cylinders only. A comparative study of these curves shows that as the radius increases the results approach close to the results for a semi-infinite medium [19]. Thus for very thick cylinders, the planar approximation is valid.

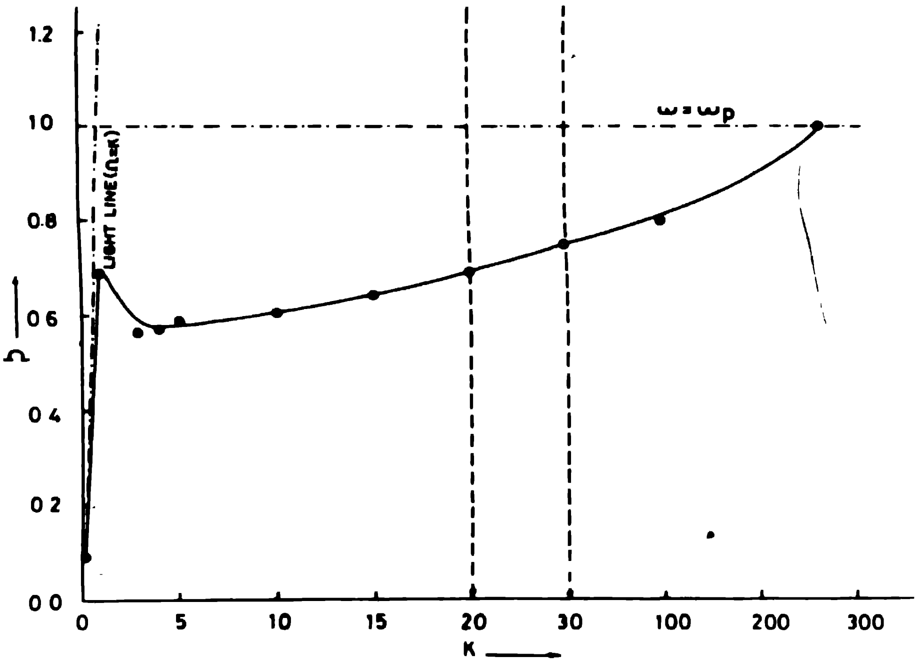

Figure 2. Dispersion curve for surface plasmon-polaritons with spatial dispersion for InAs cylinders (n = 1, R = 0.01) in vacuum (vertical broken lines denote scale change)

The dispersion curve with spatial dispersion [eq. (22)] has been shown in Figure 2 for InAs for n = 1, R = 0.01, $\xi = \frac{\beta}{c}$ = 1.97 × 10^{-4} taking the bounding medium to be vacuum *i.e.* ε_B = 1. It also shows the existence of a minimum.

3. Conclusions

The dispersion curves without spatial dispersion show that the energy of the surface plasmon-polariton mode is localized at the surface and it cannot 'leak' into the interior of the crystal as the coupled mode frequency always lies below the bulk plasma frequency *i.e.* Ω = 1 or $\omega = \omega_p$. If the plasma is collisionless, as in the present case, then the surface polariton mode propagates along the surface without decay. A comparison of Figures 1 and 2 for same values of n and R, shows that as a result of spatial dispersion, the non-radiative surface plasmon-polariton mode crosses the bulk plasma frequency at around $K = 270$. As a consequence, the energy of the surface mode leaks into the bulk of the polar semiconductor medium, leading to finite lifetime of surface polaritons even in a collisionless plasma system. The mode is still non-radiative as it does not cross the light line anywhere and is confined to the region $\Omega < K$ as before.

The non-radiative surface modes of cylinders should be detectable in an attenuated total reflection (ATR) experiment ; however to the author's knowledge, no such experimental investigation has so far been performed for the cylindrical geometry.

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